Q11 – 1. Schulaufgabe aus der Mathematik – 15.01.2010

1. (a) $f'(x) = 2x\cos(x^2)$

   (b) $g'(x) = 2\sin x\cos x (= \sin(2x))$

   (c) $h'(x) = 2x\cos x - x^2\sin x = x\,(2\cos x - x\sin x)$

2. (a) $D_f = \mathbb{R} \setminus \{2\}$, Nullstelle: $x_0 = 0$

$$\begin{array}{l} (\quad x^2) \div (4x-8) = \tfrac{1}{4}x + \tfrac{1}{2} + \dfrac{4}{4x-8} \\ \underline{-x^2 + 2x} \\ \qquad 2x \\ \quad \underline{-2x + 4} \\ \qquad\quad 4 \end{array}$$

   (b) $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x}{4 - \frac{8}{x}} = \pm\infty$

$$\lim_{x\to 2^\pm} f(x) = \lim_{h\to 0^+} f(2\pm h) =$$

$$= 1 + \lim_{h\to 0^+} \frac{1}{2 \pm h - 2} = \pm\infty$$

schräge Asympt.: $y = \dfrac{1}{4}x + \dfrac{1}{2}$

Senkrechte Asympt.: $x = 2$

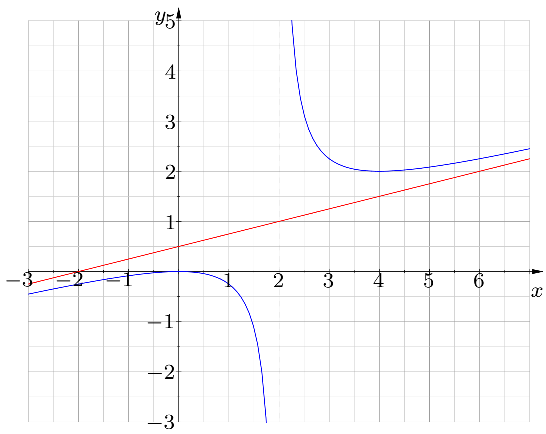
   (c) $f'(x) = \dfrac{2x(4x-8) - x^2\cdot 4}{(4x-8)^2} = \dfrac{4x^2 - 16x}{(4x-8)^2} = \dfrac{4x(x-4)}{16(x-2)^2} = \dfrac{x(x-4)}{4(x-2)^2}$

   $f'(x) = 0 \quad\Longrightarrow\quad x_{11} = 0, \quad x_{12} = 4$

   (d)

x	$f(x)$
$-3{,}0$	$-0{,}450$
$-2{,}0$	$-0{,}250$
$-1{,}0$	$-0{,}08\overline{3}$
$0{,}0$	$0{,}000$
$1{,}0$	$-0{,}250$
$1{,}5$	$-1{,}125$

x	$f(x)$
$2{,}5$	$3{,}125$
$3{,}0$	$2{,}250$
$4{,}0$	$2{,}000$
$5{,}0$	$2{,}08\overline{3}$
$6{,}0$	$2{,}250$
$7{,}0$	$2{,}450$



3. $f'(0{,}6) \approx \dfrac{f(0{,}6+h) - f(0{,}6-h)}{2h} = \dfrac{2^{-0{,}6-h}\sin(0{,}6+h) - 2^{-0{,}6+h}\sin(0{,}6-h)}{2h}$

Für h wählt man am besten „halbe Taschenrechnergenauigkeit“, bei einem zehnstelligen Rechner also $h = 10^{-5}$:

h	10^{-3}	10^{-5}	exakter Wert
$f'(x) \approx$	0,2863038586	0,2863037101	0,2863037101

4. $f(x) = (1+x)^{\frac{1}{3}}, \quad f'(x) = \dfrac{1}{3}(1+x)^{-\frac{2}{3}}, \quad f''(x) = -\dfrac{2}{9}(1+x)^{-\frac{5}{3}}, \quad f'''(x) = \dfrac{10}{27}(1+x)^{-\frac{8}{3}}$

$f(0) = 1, \quad f'(0) = \dfrac{1}{3}, \quad f''(0) = -\dfrac{2}{9}, \quad f'''(0) = \dfrac{10}{27}$

$$\sqrt[3]{1+x} \approx T_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 = 1 + \frac{1}{3}\,x - \frac{1}{9}\,x^2 + \frac{5}{81}\,x^3$$

$$T_3(0{,}03) = \underbrace{1 + 0{,}01 - 0{,}0001}_{1{,}0099} + \frac{5}{3}\cdot 10^{-6} = 1{,}009901667$$

Taschenrechnerwert: $y = 1{,}009901634, \quad \delta_{\text{rel}} = \dfrac{T_3(0{,}03) - y}{y} = 3{,}23\cdot 10^{-8}$